7. T. A. Malinovskaya, "Principles of filtration and dehydration in separation of highly dispersed suspensions," Teor. Osn. Khim. Tekhnol., 9, No. 1, 76-81 (1975).
8. W. Gregor and B. Scarlet, "Three-dimensional filtration and its importance," Filtrat. Separat., 11, No. 2, 1951-1955 (1974).
9. M. N. Markova, "Seminar on filtration problems," Teor. Osn. Khim. Tekhnol., 12, No. 5, 791-793 (1978).
10. A. Rushton, M. Shirato, F. M. Tiller, et al., "The crisis in solid—fluid separation technology," Filtrat. Separat., 10, No. 6, 670-671 (1973).
11. P. G. Romankov, "Hydromechanical processes in chemical technology," Teor. Osn. Khim. Tekhnol., 6, No. 6, 855-871 (1972).
12. M. G. Bernadiner and V. M. Éntov, Hydrodynamic Theory of Anomolous Liquid Filtration [in Russian], Nauka, Moscow (1975).
13. E. Hinch and L. Lil, "Equations of state in suspension mechanics," in: Hydrodynamic Interaction of Particles in Suspensions [Russian translation], Mir, Moscow (1980), pp. 194-215.
14. M. Reiner, Deformation and Flow [in Russian], Gostoptekhizdat, Moscow (1963), pp. 308-325.
15. I. A. Vainshtein, V. I. Konvisar, L. D. Klenysheva, et al., "Effect of salt content on the kinetics of magnetite formation, magnetic susceptibility, and mechanical dehydration," Zh. Prikl. Khim., 55, No. 1, 133-138 (1982).
16. I. A. Vainshtein, G. A. Kudenko, A. I. Babanina, et al., "Dehydration of gravel precipitate residues," Chern. Metallurg., No. 15, 76-78 (1981).
17. V. A. Zhuzhikov and I. I. Tsyrkin, "Expanding the limits of the generalized filtration equation," Teor. Osn. Khim. Tekhnol., 14, No. 3, 471-473 (1980).
18. J. Happel and G. Brener, Hydrodynamics at Low Reynolds Numbers [Russian translation], Mir, Moscow (1976), pp. 447-468.

COMBINING ASYMPTOTIC EXPANSIONS IN PROBLEMS OF THE FILTRATION OF A GAS—CONDENSATE MIXTURE

M. B. Panfilov

UDC 622.276.031:532.5.001

The asymptote-combination method is used in the problem of the startup of a gas—condensate borehole in a porous bed, which is characterized as a singularly perturbed problem. An analytical solution is constructed.

Methods of singular perturbation, which have been intensely developed in recent years, offer great possibilities for the solution of nonlinear problems of filtration theory [1-4]. Primarily, this involves the problem of the filtration of a mixture of several fluids with phase transformations, when the process is determined by many factors, and their relative role differs in different regions of motion. The solutions of such problems include sharp transitions in narrow intervals (boundary layers).

In the present work, the possibilities of the combined-asymptote method in filtration theory are investigated for gas—condensate systems.

1. Formulation of the Problem of Startup of a Gas—Condensate Borehole. The process is investigated within the framework of a modified binary model [5, 6], in which it is assumed that the weight concentrations of the components in the liquid phase are constant

$$m\frac{\partial}{\partial t}[\rho_l s+\rho_g(1-s)]+\operatorname{div}(\rho_l\vec{V}_l+\rho_g\vec{V}_g)=0, \tag{1}$$

I. M. Gubkin Institute of the Petrochemical and Gas Industry, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 4, pp. 608-616, October, 1983. Original article submitted June 13, 1982.

$$\underbrace{m\frac{\partial s}{\partial t}}_{\mathrm{I}} = \frac{\rho_g}{\rho_g(p_{bc})}\frac{dq}{dp}\left[\underbrace{\vec{V}_g \operatorname{grad} p}_{\mathrm{II}} + \underbrace{m(1-s)\frac{\partial p}{\partial t}}_{\mathrm{III}}\right] - \underbrace{\operatorname{div}\vec{V}_l}_{\mathrm{IV}}, \tag{2}$$

$$\vec{V}_l = -\frac{k\bar{k}_l(s)}{\mu_l}\operatorname{grad} p,\ \vec{V}_g = -\frac{k\bar{k}_g(s)}{\mu_g}\operatorname{grad} p. \tag{3}$$

It follows from Eq. (2) that the change in saturation (term I) is determined by the phase transition when the gas reaches a point with a new pressure value (II), the change in pressure over time (III), and the motion of the condensate itself (IV).

In addition, it is assumed that the viscosity of the two phases and the density of the liquid phase are constant, and that $\rho_g = 2\gamma p$, $\gamma = \mathrm{const}$; $q(p) = \Lambda(p_{bc}^2 - p^2)$, $\Lambda \equiv q_{mc}/(p_{bc}^2 - p_{mc}^2)$ when $p \in [p_{mc}, p_{bc}]$; $\bar{k}_g(s) = (1-s)$, $\bar{k}_l(s) = s^\beta$, where α, $\beta = \mathrm{const}$; $\beta > 2$.

The boundary and initial conditions of the problem of startup with constant mass flow rate G of a borehole in a homogeneous axisymmetric bed of power h with constant pressure at the contour and an initial unperturbed state for Eqs. (1-3) in cylindrical coordinates take the form

$$p(r,0) = p_0,\ s(r,0) = 0,\ p(r_c,t) = p_0,\ s(r_c,t) = 0,$$
$$\left\{\frac{2\pi kh\gamma}{\mu_g} r\frac{\partial p^2}{\partial r}\left[(1-s)^\alpha + \frac{\rho_l\mu_g s^\beta}{2\gamma\mu_l p}\right]\right\}_{r=r_w} = G.$$

As $r_w \to 0$, $r_c \to \infty$, the problem is self-similar and may be formulated as follows:

$$2\sqrt{T}\{[(1-s)^\alpha + \lambda_1\lambda_2 s^\beta T^{-1/2}]\xi T'\}' = -\xi^2(1-s)T' - 2\xi^2 s'(T - \lambda_1\sqrt{T}), \tag{4}$$

$$-\xi^2 s' = 2\lambda_3\xi(1-s)^\alpha(T')^2 + 2\lambda_3\xi(1-s)\sqrt{T}T' + \lambda_2(\xi s^\beta T^{-1/2}T')', \tag{5}$$

$$T(\infty) = 1,\ s(\infty) = 0, \tag{6}$$

$$\lim_{\xi\to 0}\xi T'[(1-s)^\alpha + \lambda_1\lambda_2 s^\beta T^{-1/2}] = \varepsilon, \tag{7}$$

where $T(\xi) = p^2/p_0^2$; $\xi^2 = r^2/\varkappa t$; $\lambda_1 = \rho_l/\rho_g(p_0)$; $\lambda_2 = \mu_g/\mu_l$; $\lambda_3 = {}^1/{}_2\Lambda p_0^2$; $\varepsilon = G\mu_g/\pi khp_0\rho_g(p_0)$; $\varkappa = kp_0/m\mu_g$. It is assumed that $p_0 = p_{bc}$.

On account of the presence of the radical $\sqrt{T}$, the problem is incorrectly posed, since there does not exist a solution satisfying the condition in Eq. (7) at the point $\xi = 0$ [7]. The Barenblatt "expanding-borehole" method may be used for regularization. A different approach is used here: the radical $\sqrt{T}$ is always understood to mean the partial sum of its binomial expansion

$$\sqrt{T} = \sqrt{1 + (T-1)} \sim \sum_{k=0}^{N}\binom{0.5}{k}(T-1)^k,\ N < \infty.$$

This allows the condition in Eq. (7) to be retained at the same point $\xi = 0$.

In practice, the parameter ε is always small ($\varepsilon \sim 10^{-5}$-10^{-1}), which allows the asymptote of the solution to be investigated as $\varepsilon \to 0$. The simplest regular small-parameter method was used earlier in analogous problems of single-phase filtration [8, 9] and also two-phase filtration without phase conversions [10]. In the presence of phase transition, the problem is singularly perturbed.

2. External Expansion. The expansion of the solution suitable in the external region where $\xi \gg 0$ is obtained by the direct use of the small-parameter method. Assume that

$$T(\xi;\varepsilon) = 1 + \sum_{k=1}^{\infty}\varepsilon^k T_c(\xi),\ s(\xi;\varepsilon) = \sum_{k=1}^{\infty}\varepsilon^k s_c(\xi).$$

The boundary conditions take the form

$$T_c(\infty) = 0,\ s_c(\infty) = 0,\ \forall k; \tag{8}$$

$$\xi T_1'|_{\xi=0}=1,\ [\xi T_2'-\alpha s_1 T_1']_{\xi=0}=0, \text{ etc.} \tag{9}$$

Substitution into Eqs. (4)-(7) leads to a linear system for each pair (T_c, s_c). The form of the expansion satisfying Eq. (8) is as follows:

$$T(\xi)=1+{}^1/_2C_1\varepsilon\mathrm{Ei}(-\omega\xi^2)+{}^1/_4\varepsilon^2\lambda_3C_1^2\{(\lambda_1-\alpha-1)\mathrm{Ei}^2(-\omega\xi^2)+$$
$$+4(\alpha-1)\mathrm{Ei}(-2\omega\xi^2)+2[2C_2-(\alpha-1)\exp(-\omega\xi^2)]\mathrm{Ei}(-\omega\xi^2)\}+O(\varepsilon^3), \tag{10}$$

$$s(\xi)=-\varepsilon C_1\lambda_3\mathrm{Ei}(-\omega\xi^2)+\varepsilon^2\lambda_3C_1^2\{\exp(-2\omega\xi^2)\xi^{-2}-{}^1/_2\lambda_3(\lambda_1-\alpha)\mathrm{Ei}^2(-\omega\xi^2)+$$
$$2[\omega-\lambda_3(\alpha-1)]\mathrm{Ei}(-2\omega\xi^2)+\lambda_3[(\alpha-1)\exp(-\omega\xi^2)-2C_2]\mathrm{Ei}(-\omega\xi^2)\}+O(\varepsilon^3), \tag{11}$$

where Ei(u) is an integral exponential; $\omega = 1/4[1-4\lambda_3(\lambda_1-1)]$.

All the constants C_i cannot be determined from the second boundary condition in Eq. (9). In particular, the second term in Eq. (10) does not satisfy this condition with any choice of the constant C_2. This means that the expansion constructed does not converge to the precise solution in the vicinity of the point $\xi = 0$.

It may be shown that for the true solution $\xi = 0$ is a branch point: $1 - s \sim \xi^{\varepsilon}$; for the external expansion in Eq. (11), on the other hand, it is a pole [11]. The type of singularity is variable. Such problems are characterized by the presence of the complex $\xi(\xi^k+\varepsilon)$, $k>1$ in front of the principal derivative rather than the small parameter ε, as in the classical case [1, 3], or the factor $(\xi + \varepsilon)$, as in the Laitkhill problem [4].* Therefore, when $\varepsilon = 0$, instead of a reduction in order of the equation or a shift in the singularity from the point $\xi =-\varepsilon$ to the point $\xi = 0$, there is a degeneration of singularity type at the point $\xi = 0$. The method of combining asymptotic expansions may be used to solve such problems.

3. Transition to Internal Variables. The next step is to construct the internal expansion which is the asymptote of the accurate solution in regions of small ξ, such that $\xi = O(\Delta(\varepsilon))$, where $\Delta \to +0$ as $\varepsilon \to +0$. In the new region ξ is replaced by the variable $\eta = \xi/\Delta$, which is independent of ε. The desired functions are written in the form

$$T_*(\eta;\ \varepsilon)\equiv T(\eta\Delta;\ \varepsilon)=1+\sum_{i=1}^{\infty}a_i(\varepsilon)T_{*i}(\eta), \tag{12'}$$

$$s_*(\eta;\ \varepsilon)\equiv s(\eta\Delta;\ \varepsilon)=\sum_{i=1}^{\infty}b_i(\varepsilon)s_{*i}(\eta). \tag{12''}$$

Here $a_{i+1} = o(a_i)$; $b_{i+1} = o(b_i)$; $a_1 = o(1)$; $b_1 = o(1)$.

It is required to determine $\Delta(\varepsilon)$ and the sequences $\{a_i\}$ and $\{b_i\}$. The initial problem in the new variables takes the form

$$2\sqrt{T_*}\{\eta T_*'[(1-s_*)^{\alpha}+\lambda_1\lambda_2s_*^{\beta}T_*^{-1/2}]\}'=-[(1-s_*)T_*'+2s_*'(T_*-\lambda_1\sqrt{T})]\eta^2\Delta^2, \tag{13}$$

$$\underbrace{-\eta^2s_*'\Delta^2}_{\mathrm{I}}=\underbrace{2\lambda_3\eta(1-s_*)^{\alpha}(T_*')^2}_{\mathrm{II}}+\underbrace{2\lambda_3\eta(1-s_*)\sqrt{T_*}T_*'\Delta^2}_{\mathrm{III}}+\underbrace{\lambda_2(\eta s_*^{\beta}T_*^{-1/2}T_*')'}_{\mathrm{IV}}, \tag{14}$$

$$\lim_{\eta\to 0}\eta T_*'[(1-s_*)^{\alpha}+\lambda_1\lambda_2s_*^{\beta}T_*^{-1/2}]=\varepsilon. \tag{15}$$

The external conditions for the internal problem disappear.

The form of the first terms of the sequence $\{a_i\}$ is found from the boundary conditions and the combination conditions. To determine a_1, the external expansion of $T(\xi)$ in terms of the variable η is written

$$T(\eta\Delta;\ \varepsilon)=1+C_1\varepsilon\ln\Delta(\varepsilon)+{}^1/_2C_1\varepsilon(\ln\omega\eta^2+C_e)+\ldots,$$

and hence it follows that a logarithmic term must appear in the internal expansion

*This representation of Eq. (5) is obtained upon eliminating the function T (ξ).

$$a_1(\varepsilon) = O(\varepsilon \ln \Delta(\varepsilon)), \text{ while } \quad T_{*1}(\eta) = \text{const.} \tag{16}$$

Now a_2 is found from Eq. (15):

$$a_2(\varepsilon) = \varepsilon. \tag{17}$$

The determination of $\Delta(\varepsilon)$ and $\{b_i\}$ is a nodal point in the solution of the problem. There are purely mathematical methods of finding these quantities, based on the analysis of the degeneration of the singularities of the solution [11]. However, in applied problems these methods may be successfully replaced by certain a priori physical considerations.

Consider Eq. (14), where the numbering of terms is as in Eq. (2). In the internal region, i.e., close to the borehole, the velocity and pressure gradient are large; therefore, it may be assumed that the change in saturation will be determined primarily by convective accumulation of the condensate (term II) and its partial removal from the bed (IV). These are the principal terms of the equation in the internal region. Term I is also a principal term, since otherwise the saturation distribution will be steady, which is an extremely particular case. Therefore, in the general case, two relations are obtained:

$$s'_* \Delta^2 = O((1 - s_*)^\alpha (T'_*)^2) = O((s_*^\beta T_*^{-1/2} T'_*)').$$

Substituting from Eq. (12) and taking account of Eqs. (16) and (17), it follows that

$$\Delta(\varepsilon) = \varepsilon^{1-1/2\beta}, \; b_1(\varepsilon) = \varepsilon^{1/\beta}. \tag{18}$$

It follows from Eqs. (16) and (18) that $a_1(\varepsilon) = \varepsilon \ln \varepsilon$. Simple switching of variants leads further to the conclusion that $a_3(\varepsilon) = \varepsilon^{1+1/\beta}$.

4. Internal Expansion. Substitution of the results obtained into Eqs. (13), (14) and further refinement of the terms of the asymptotic sequence lead to the expressions

$$T_*(\eta) \sim 1 + D_1 \varepsilon \ln \varepsilon + \varepsilon (D_2 \ln \eta + B_2) + \varepsilon^{1+1/\beta} \left(\alpha D_2 \int \frac{s_{*1} d\eta}{\eta} + D_3 \ln \eta + B_3 \right) + \varepsilon^{1+2/\beta} T_4(\eta),$$

$$s_*(\eta) \sim \varepsilon^{1/\beta} s_{*1}(\eta) + \varepsilon^{2/\beta} s_{*2}(\eta),$$

where the function s_{*1} of the first approximation is found from the formula

$$s'_{*1}(\eta) = -\frac{2\lambda_3 D_2}{\eta^3 + \lambda_2 \beta D_2 \eta s_{*1}^{\beta-1}}, \tag{19}$$

and the functions s_{*2} and T_{*4} of the second approximation are found from the system

$$(\eta T'_{*4})' = \alpha D_2 [{}^1/_2 (\alpha + 1)(s_{*1}^2)' + s'_{*2}] + \alpha D_3 s'_{*1}, \tag{20'}$$

$$s'_{*2}(\eta^2 + \lambda_2 \beta D_2 s_{*1}^{\beta-1}) + \lambda_2 D_2 (\beta - 1)(s_{*1}^\beta)' s_{*2} + \lambda_2 D_2 \alpha (s_{*1}^{\beta+1})' + \lambda_2 D_3 (s_{*1}^\beta)' + 2\lambda_3 D_2 (\alpha D_2 s_{*1} + 2D_3)/\eta = 0. \tag{20''}$$

The constant $D_2 = 1$ is found from the internal condition in Eq. (15). The remaining constants and conditions for the differential equations are found by combination with the external expansion. The combination technique may be illustrated as follows. Introducing the "mean" coordinate x, which is of the order of unity in the region of overlap of the expansions

$$x = \xi/\delta(\varepsilon) = \eta \Delta(\varepsilon)/\delta(\varepsilon), \text{where } \delta = o(1) \text{ and } \Delta = o(\delta(\varepsilon)) \quad \text{as } \varepsilon \to 0,$$

the expansion is written in terms of the new variable, and represented in the form of asymptotic series as $\varepsilon \to 0$, with x fixed:

$$T(x\delta;\ \varepsilon) = 1 + C_1 \varepsilon \ln \delta + {}^1/_2 C_1 \varepsilon (\ln \omega x^2 + C_e) + \ldots,$$

$$T_*(x\delta/\Delta;\ \varepsilon) = 1 + \varepsilon \ln \delta + [D_1 - (1 - {}^1/_2 \beta)] \varepsilon \ln \varepsilon + \varepsilon (\ln x + B_2) + \ldots$$

In the region where the expansions overlap, they coincide, and therefore $C_1 = 1$, $D_1 = 1 - {}^1/_2 \beta$, $B_2 = {}^1/_2 (\ln \omega + C_e)$.

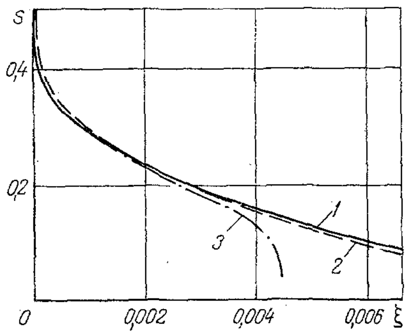

Fig. 1. Internal expansion of the function s(ξ): 1) second approximation; 2) first approximation; 3) asymptote of the first approximation as $\xi \to 0$.

The combination procedure for the function s is somewhat complicated in view of the absence of an explicit expression for s in the internal region. A representation for the function s_{*1} in terms of the mean variable is found as follows. The derivative in Eq. (19) is written in terms of the variable x, and the resulting expression is regrouped in the form of an asymptotic series as $\varepsilon \to 0$, with fixed x, and then integrated. The functions s_{*2}, T_{*4} are dealt with analogously. Finally, combination allows the constants $D_3 = B_3 = 0$ to be found and boundary conditions to be established for Eqs. (19), (20):

$$s_{*1}(\infty) = 0,\ s_{*2}(\infty) = 0,\ T_{*4}(\infty) = 0,\ \eta T'_{*4}|_{\eta=\infty} = 0, \tag{21}$$

producing uniqueness of the functions s_{*1}, s_{*2}, and T_{*4}.

The problem in Eqs. (19), (21) for s_{*1} is solved in quadratures:

$$\lambda_3 \eta^{-2} = \int_0^{s_{*1}} \exp\left[\frac{\lambda_2}{\lambda_3}(s_{*1}^{\beta} - u^{\beta})\right] du, \tag{22}$$

while s_{*2} and T_{*4} are found numerically.

As $\eta \to 0$, the following expression may be found from Eq. (22) for s_{*1}:

$$s_{*1} \simeq \left\{\frac{\lambda_3}{\lambda_2} \ln\left[\frac{\beta\lambda_3}{\eta^2 \Gamma(1/\beta)} \left(\frac{\lambda_2}{\lambda_3}\right)^{1/\beta}\right]\right\}^{1/\beta},$$

where Γ is an Euler gamma function.

The form of the internal expansion is shown in Fig. 1.

5. Intermediate Expansion. Analysis of the combination procedure shows that the first internal approximation only overlaps with the second external approximation, and there is no region where the first approximations of both expansions overlap (see Fig. 3). This means that there is some intermediate region where the true solution has an expansion that differs from the internal and external expansions even in the first term. It is constructed using the new coordinate $\zeta = \xi/\Delta_1(\varepsilon)$, where $\Delta_1 \to +0$ as $\varepsilon \to +0$, and the new dependent functions $T^*(\zeta;\ \varepsilon) = T(\zeta\Delta_1;\ \varepsilon)$, $s^*(\zeta;\ \varepsilon) = s(\zeta\Delta_1;\ \varepsilon)$.

The initial system of equations in the new variables takes the form in Eqs. (13)-(15) with the corresponding change in notation. The expansions for the functions T^* and s^* are sought in a form analogous to Eq. (12). For α_1, α_2, and also T_1^*, as before, Eqs. (16) and (17) remain in force. It follows from Eq. (11) that a logarithmic term may appear in the expansion of s^*:

$$b_1(\varepsilon) = O(\varepsilon \ln \Delta_1(\varepsilon)), \text{ while } \quad s_1^*(\zeta) = \text{const}. \tag{23}$$

Terms of the sequence $\{b_i\}$ and Δ_1 are again found from additional a priori considerations on the physics of the process.

The change in saturation in the external region is determined solely by term III in Eq. (14) and in the internal region by terms II, IV. Therefore, in the intermediate region the principal terms should be III (which is combined with the external expansion), II (which is combined with the internal expansion), and I (or else the sum of the two nonlinear terms II and III of the same sign will be equal to zero, which is impossible). Therefore,

$$(s^*)'\Delta_1^2 = O((T^{*\prime})^2) = O(T^{*\prime}\Delta_1^2),$$

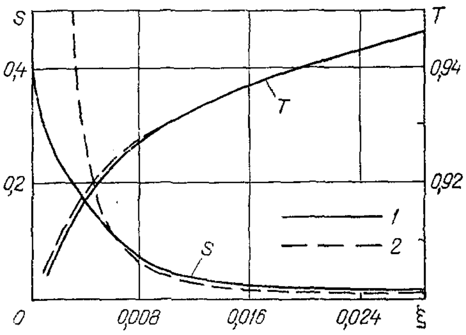

Fig. 2. Intermediate expansion of the solution of the problem: 1) second approximation; 2) first approximation.

and hence

$$\Delta_1(\varepsilon)=\sqrt{\varepsilon},\ b_2(\varepsilon)=\varepsilon. \tag{24}$$

It is evident from Eq. (23) that $b_1(\varepsilon) = \varepsilon\ln\varepsilon$.

Substituting Eq. (24) into Eqs. (13) and (14), subsequent analysis of the sequences $\{a_i\}$ and $\{b_i\}$ and two-sided combination allows the first approximation of the intermediate expansion to be obtained in the form

$$T^*(\zeta)=1+{}^1/_2\varepsilon\ln\varepsilon+{}^1/_2\varepsilon(2\ln\zeta+\ln\omega+C_e)+O(\varepsilon^2\ln^2\varepsilon),$$
$$s^*(\zeta)=-\lambda_3\varepsilon\ln\varepsilon+{}^1/_3\varepsilon(\zeta^{-2}-\ln\omega\zeta^2-C_e)+O(\varepsilon^2\ln^2\varepsilon).$$

The second approximation, including terms of order $\varepsilon^2\ln^2\varepsilon$, $\varepsilon^2\ln\varepsilon$ and ε^2, is not given in view of its complexity.

The form and convergence of the intermediate expansion is illustrated in Fig. 2.

6. Final Dependences for the Distributions of p and s. In the problem three regions are distinguished; in each one it is sufficient to use the first approximation. Passing to the variable ξ, the first approximations of all the expansions are found to be as follows: external region $\xi = O(1)$:

$$p^2/p_0^2\sim 1+{}^1/_2\varepsilon\mathrm{Ei}(-\omega\xi^2),\ s\sim-\varepsilon\lambda_3\mathrm{Ei}(-\omega\xi^2),$$

intermediate region $\xi = O(\sqrt{\varepsilon})$:

$$p^2/p_0^2\sim 1+{}^1/_2\varepsilon(\ln\omega\xi^2+C_e),\ s\sim\frac{\varepsilon^2\lambda_3}{\xi^2}-\varepsilon\lambda_3(\ln\omega\xi^2+C_e);$$

internal region $\xi = O(\varepsilon^{1-1/2\beta})$.

$$p^2/p_0^2\sim 1+{}^1/_2\varepsilon(\ln\omega\xi^2+C_e)-\alpha\varepsilon\int\limits_{\xi}^{\infty}\frac{s(u;\ \varepsilon)}{u}\,du, \tag{25}$$

$$\frac{\lambda_3\varepsilon^2}{\xi^2}\sim\int\limits_0^s\exp\left[\frac{\lambda_2}{\lambda_3\varepsilon}(s^\beta-u^\beta)\right]du. \tag{26}$$

As $\xi\to 0$, Eqs. (25), (26) are simplified:

$$p^2/p_0^2\sim 1+{}^1/_2\varepsilon(\ln\omega\xi^2+C_e)-\frac{\alpha\beta}{2(\beta+1)}\left(\frac{\lambda_3}{\lambda_2}\right)^{1/\beta}\left\{\varepsilon\ln\left[\frac{\lambda_3\beta\varepsilon^2}{\Gamma(1/\beta)\xi^2}\left(\frac{\lambda_2}{\lambda_3\varepsilon}\right)^{1/\beta}\right]\right\}^{1/\beta+1},$$

$$s\sim\left\{\varepsilon\frac{\lambda_3}{\lambda_2}\ln\left[\frac{\lambda_3\beta\varepsilon^2}{\Gamma(1/\beta)\xi^2}\left(\frac{\lambda_2}{\lambda_3\varepsilon}\right)^{1/\beta}\right]\right\}^{1/\beta}.$$

The convergence and overlapping of all the expansions is shown in Fig. 3. Data on the Karadag deposit are used in the calculations [12].

The first approximation differs from the second by no more than 2.2% for the saturation and even less for the pressure.

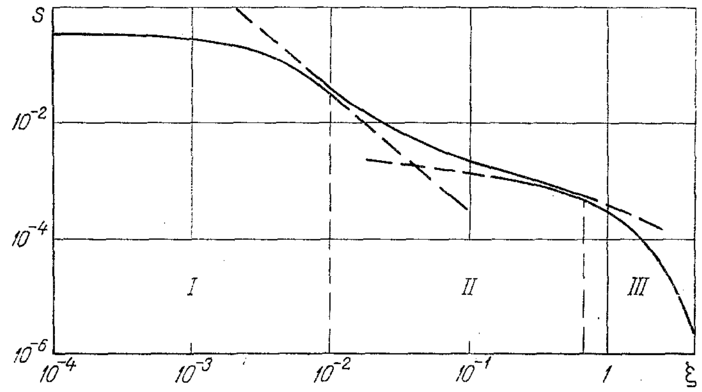

Fig. 3. Overlapping of expansions of the function $s(\xi)$ (first approximations): I) internal; II) intermediate; III) external region.

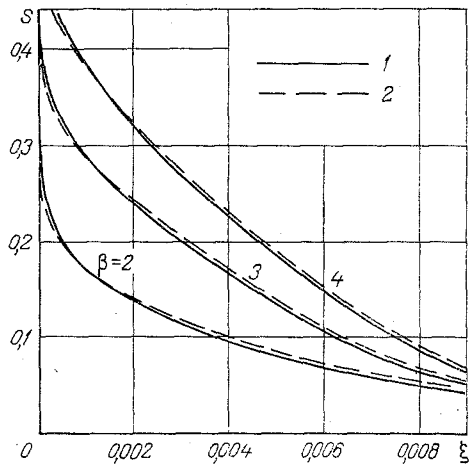

Fig. 4. Influence of the form of the curves of phase permeability on the distribution of condensate saturation: 1) $\alpha = 1$; 2) 2.

7. Analysis of the Solutions. In the external region, the condensate saturation is negligibly small; nevertheless, the influence of the condensation process on filtration of the gas exists even in this region and is expressed in that the pressure falls more rapidly than in the absence of phase transitions. This happens because when some of the gas phase is converted to liquid, it is compressed and, correspondingly, the remainder of the gas phase expands, leading to additional reduction in pressure. Formally, it is reflected in the appearance of the factor $\omega \leqslant 1/4$ in the piezoconduction coefficient.

In the intermediate region the condensate saturation reaches perceptible values, but in fact only the gas phase moves. The accumulation of condensate in pores occurs not only because of the fall in pressure over time, but also because of the pressure gradients forcing the gas to the points of lower pressure associated with condensation. This process may be called convective mass transfer. It obviously differs significantly from the quasistatic process occurring in a PVT bomb.

In the internal region both phases move. The intensity of convective mass transfer is higher than the rate of condensate removal from the plate, which leads to sharp rise in saturation on leaving the intermediate region. However, the increase in saturation over time is slow.

The parameters α and β influence the pressure distribution equally. The distribution of the saturation is influenced significantly by the form of the curve $\bar{k}_l(s)$, but is practically independent of the form of the curve $\bar{k}_g(s)$: In the first approximation, Eq. (26), the parameter α does not appear at all (Fig. 4).

It follows from the solutions obtained that with sufficient accuracy the motion of the gas phase over the whole region may be described by the equation

$$\frac{\partial p}{\partial t}=\frac{\tilde{\varkappa}}{r}\frac{\partial}{\partial r}\left[r\,\bar{k}_g(s)\,\frac{\partial p^2}{\partial r}\right],$$

where $\tilde{\varkappa}\equiv\varkappa/4\omega$ is the modified piezoconduction coefficient, which now increases with increase in condensation intensity λ_3 and density ratio λ_1.

NOTATION

p, pressure; ρ, density; μ, dynamic viscosity; q(p), isotherm of contact condensation of the bed mixture (volume ratio of the liquid phase in a PVT bomb at pressure p and the system at p_{bc}); s, condensate saturation of the rock space; T, dimensionless square of the pressure; m, porosity; k, bed permeability; $\bar{k}(s)$, relative phase permeability; **V**, filtration-rate vector; ε, dimensionless borehole output (small parameter): α, β, parameters of the phase-permeability curves for gas and liquid, respectively; ϰ, piezoconduction of bed; t, time; r, radial coordinate; ξ, ζ, η, self-similar external, intermediate, and internal independent variables; C_e = 0.5772..., Euler constant. Subscripts: g, gas; l, liquid; bc, beginning of condensation; mc, maximum condensation; w, borehole wall; c, external contour of bed; 0, initial state; superscript asterisk denotes intermediate region; subscript asterisk denotes internal region.

LITERATURE CITED

1. A. B. Vasil'eva and V. F. Butuzov, Asymptotic Expansion of the Solution of Singularly Perturbed Equations [in Russian], Nauka, Moscow (1973).
2. P. A. Lagerstrom and R. G. Kasten, "Basic concepts of the method of singular perturbations," in: Mechanics [Russian translation], Vol. 2, IL (1973), No. 138, pp. 50-109.
3. S. A. Lomov, Introduction to the General Theory of Singular Perturbations [in Russian], Nauka, Moscow (1981).
4. A.-H. Nayfeh, Perturbation Methods, Wiley (1973).
5. N. Kh. Magerramov and A. Kh. Mirzadzhanzade, "Filtration of gas–condensate mixtures in a porous medium," Prikl. Mat. Mekh., 24, No. 6, 1094-1099 (1960).
6. M. B. Panfilov, Modification of a Binary Model of the Filtration of a Gas–Condensate Mixture [in Russian], Moscow (1982); deposited at VNIIÉgazprom March 3, 1982, No. 478G3-D82.
7. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of Nonsteady Filtration of liquid and Gas [in Russian], Nedra, Moscow (1972).
8. S. N. Buzinov and I. D. Umrikhin, Researches into Strata and Pores with Flexible Regime of Filtration [in Russian], Nedra, Moscow (1964).
9. P. Ya. Polubarinova-Kochina, Theory of Groundwater Motion [in Russian], Nauka, Moscow (1977).
10. H. Pascal and I. Spulber, "One some problems of nonsteady multiphase flow through porous media," Rev. Roum. Sci. Tech., Ser. Mec. Appl., 22, No. 4, 485-495 (1977).
11. M. B. Panfilov, Solving the Problem of Filtration of Gas–Condensate Mixture around a Borehole by the Method of Asymptotic-Expansion Combination [in Russian], Moscow (1982); deposited at VNIIÉgazprom. March 3, 1982, No. 479G3-D82.
12. A. Kh. Mirzadzhanzade et al., Development of Gas–Condensate Deposits [in Russian], Nedra, Moscow (1967).